**E-84 Electric & Magnetic Circuits & Devices Sam Tanenbaum**

**Solution Key for PS #3**

**1.** Please calculate **S, P,** and **Q** for the following series and parallel ac circuits.



**a.** When **Z1 = j2**; and **Z2 = -j.**

We use the formula **S = V•I\* = VV\*/Z\* = 104/Z\* = P + jQ**

**In series, Z = j2 - j = j,** so that

**S = 104/-j = j104 (or 104 /90º**), so that **P = 0 and Q = 104 VAR**

**In parallel, Z = Z1Z2/(Z1 + Z2) = 2/j = -j2, so that**

**S = 104/j2 = -j5,000 (or 5,000 /-90º),** so that **P = 0 and Q = -5000 VAR**

**b.** When **Z1 = 2 (1 + j); and Z2 = 1 – j.**

**For this case, in series, Z = 2 + 2j + 1 – j = 3 + j,** so that **Z\* = 3 – j**, and

**S = 104/(3 - j) = (104/10)(3 + j) (or 3,160/18.4º**), so that

**P = 3,000 W and Q = 1,000 VAR**

**In parallel, Z = Z1Z2/(Z1 + Z2) = 4/(3 + j)** so that

**S = (104/4)(3 – j) = (2,500)(3 – j) (or 7,910/-18.4º**), so that

**P = 7,500 W and Q = -2,500 VAR**

**c.** When **Z1 = 2; and Z2 = -j.**

**For this case, in series, Z = 2 - j,** so that **Z\* = 2 + j**, and

**S = 104/(2 + j) = (104/5)(2 - j) (or 4,470/-26.6º**), so that

**P = 4,000 W and Q = -2,000 VAR**

**In parallel, Z = Z1Z2/(Z1 + Z2) = -2j/(2 - j)** so that Z\* = 2j/(2 + j)

**S = (104/2j)(2 + j) = (5,000)(1 – 2j) (or 11,180/-63.4º**), so that

**P = 5,000 W and Q = -10,000 VAR**

**3.** For a standard 60 Hz (= 120 ac circuit, express the following in **phasor** notation and calculate **S, P, and Q**..

**a. v(t) = 120√2cost, i(t) = 10√2cos(t – 60º)**

**V = 120/0º, I = 10/-60º**

**S = V•I\* = 1,200/+60º VA**

**P = 1,200 cos60º = 600 W, Q = 1,200 sin60º = 1,039 VAR**

**b. v(t) = 50cost, i(t) = 10cos(t – 45º)**

**V = 35.4/0º, I = 7.07/-45º**

**S = V•I\* = 250/+45º VA**

**P = 250 cos45º = 177 W, Q = 250 sin45º = 177 VAR**

**c. v(t) = 1,000cost, i(t) = 50cos(t + 77º)**

**V = 707/0º, I = 35.4/+77º**

**S = V•I\* = 25,000/-77º VA**

**P = 25,000 cos77º = 5,620W, Q = -25,000 sin77º = 24,400 VAR**

**6.** Findthe Norton equivalent circuits for **everything except** the capacitive load in the circuit shown below, and use your results to find IC, the current in the capacitor.



**The Norton current, IN, is the value for IC when the capacitor is replaced by a short circuit. The makes the voltage zero at the node (call it A) between the 10 Ω and 20 Ω resistors on the top of the circuit, so that:**

1. **The 20 Amp current source has two paths to ground through each 10 Ω resistor. Thus 10 Amps is added to IN from this source.**
2. **The current from the voltage source = 20/20 = 1 Amp that adds to IN**

**Hence IN = 11 A.**

**The Thevenin Resistance, RTh, is the resistance from A to ground with**

* **the capacitor removed,**
* **the current source replaced by an open circuit, and**
* **the voltage source replaced by a short circuit.**

**With these changes there are two parallel paths from A to ground—one through the 20 Ω resistor and the other path through the two 10 Ω resistors.**

**Hence RTh = 20|| 20 = 10 Ω.**

**The Norton equivalent circuit has**

* **an 11 A current source (IN) which divides between the load capacitor and**
* **the 10 Ω RTh resistance in parallel with the load.**

**Using the current divider formula,**

**IC = INRTh/(Rth + ZC) = 11•10/(10 – j10) = 11/(1 – j) = 7.78/+45º**

**9.** The ac power supply shown below has **V0 = 200 V and R0 = 5 ?.** A load is attached as shown and it is found that **I = 0.4/-60º**



**a.** Is this an inductive or a capacitive load?

**Inductive (since current lags)**

**b.** Determine **S, P,** and **Q** for the **power supply**.

**S = VI\* = 200•.4/+60º = 40(1 + j√3) VA** Hence

**P = 40 Watts and Q = 69.3 VARS** for the power supply

**c.** What is the **power** delivered to the **load**?

**PLoad = P – PLoss** (where PLoss is the power taken by the 5 Ω internal resistance)

PLoss = 5II\* = 5•(.4)2 = 0.8 W. Hence

**PLoad = 40 - .8 = 39.2 W**

**d.** Show that **RL = 245 Ω** and **XL = 433 Ω.**

**There are many ways to show this. For example:**

**ZTotal = V/I = (200/.4)/+60º = 250(1 + j√3) = 250 + j433 Ω**

But **ZLoad = ZTotal - 5 = 245 + j433 Ω** so that

**RL = 245 Ω** and **XL = 433 Ω.**

**e.** Show that the **maximum power** that could be delivered to a load by this power supply is **2,000 W**.

Maximum power is delivered to the load when **ZLoad = 5 Ω** (the internal resistance of the power supply).

For this case, I = 200/10 = 20 A, so that

**PLoad = I2R= 400•5 = 2,000 W**

**10.** A manufacturer needs several 100 hp electric motors (each requiring nearly 100 kW of power) plus air conditioning, lighting, and many other smaller motors and appliances, so that her total electrical power load is **1 MW** with a power factor of .5 (lagging or inductive). If 60 Hz power is supplied at **2kV** the circuit has the form shown below:



**a.** Show that the current for this plant is **I = 103/-60º**

**P = VIcos= 2,000I(.5),** so that **I = 106/103 = 103 A**, while the **phase of I = -60º,** since cos-1.5 = 60º, and current is given as lagging.

**Hence I = 103/-60º**

**b.** What are the real and imaginary parts of the load for the plant?

**ZL =** **V/I = 2,000/(103/-60º) = 2/+60º = 1 + j√3 Ω. Hence**

**RL = 1 Ω and XL = √3 Ω**

**c.** From your results in (b) show that a shunt capacitor of **1.15 mF** will bring the power factor to unity. Hint: recall that, from the notes, the desired shunt capacitor has Zs = -jXc where Xc = XL +(RL2/XL)

To get unity power factor, we must add a shunt capacitor with

Xc = XL +(RL2/XL) = **√3 + 1/√3 = 4/√3 = 2.31 Ω**

But for a capacitor, XC = 1/C, so that

**C = 1/XC = (2π•60•2.31)-1 = .00115 F = 1.15 mF**

**d.** Repeat the calculation assuming that power is supplied **at 200 V**, and show that the required value for Cs in that case is **100 times larger.** (Physically, this is because the energy stored in a capacitor is CV2/2, so reducing the voltage by a factor of 10 requires you to increase the capacitor value by 100.)

If the power source is only **200 V,** the current (calculated as in a) is **I = 104/-60º**

Hence the load impedance (calculated as in b) is

**ZL =** **V/I = 200/(104/-60º) = .02/+60º = .01(1 + j√3) Ω. Hence**

**RL = .01 Ω and XL = .01√3 Ω.**

Since these values are reduced by a factor of 100, the reactance for the shunt capacitor is also reduced by a factor of 100, which means that the value for C (calculated as in part c) must **increase by a factor of 100—i.e.,**

**C = .115 F = 115 mF**